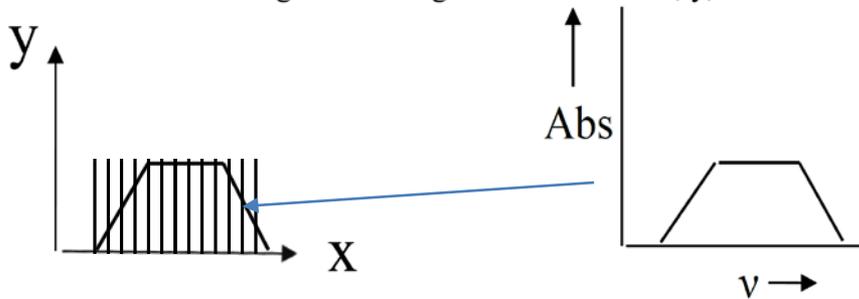
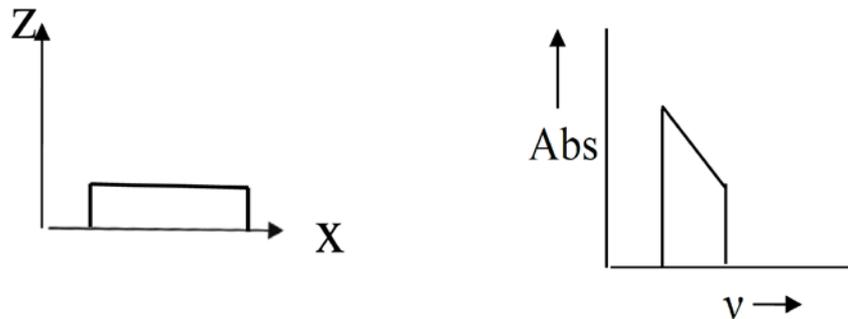


4. Given the shape depicted below shown in 3 views filled with water: draw curves that indicate the intensity of absorbed radio frequency energy during an MRI of this object when the magnetic field gradient is in the x, y, z directions.

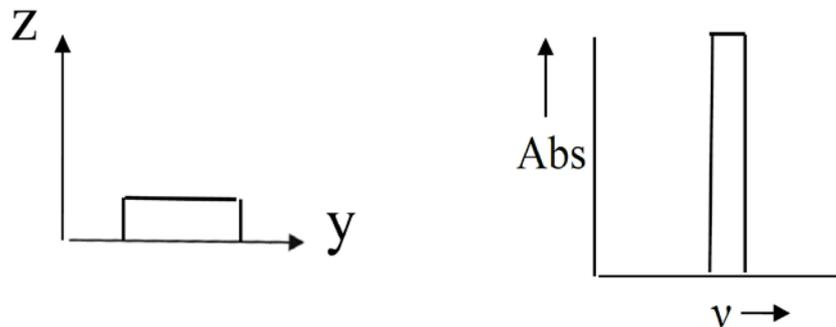
ALWAYS: slice perpendicular to gradient



Field gradient in
 x direction

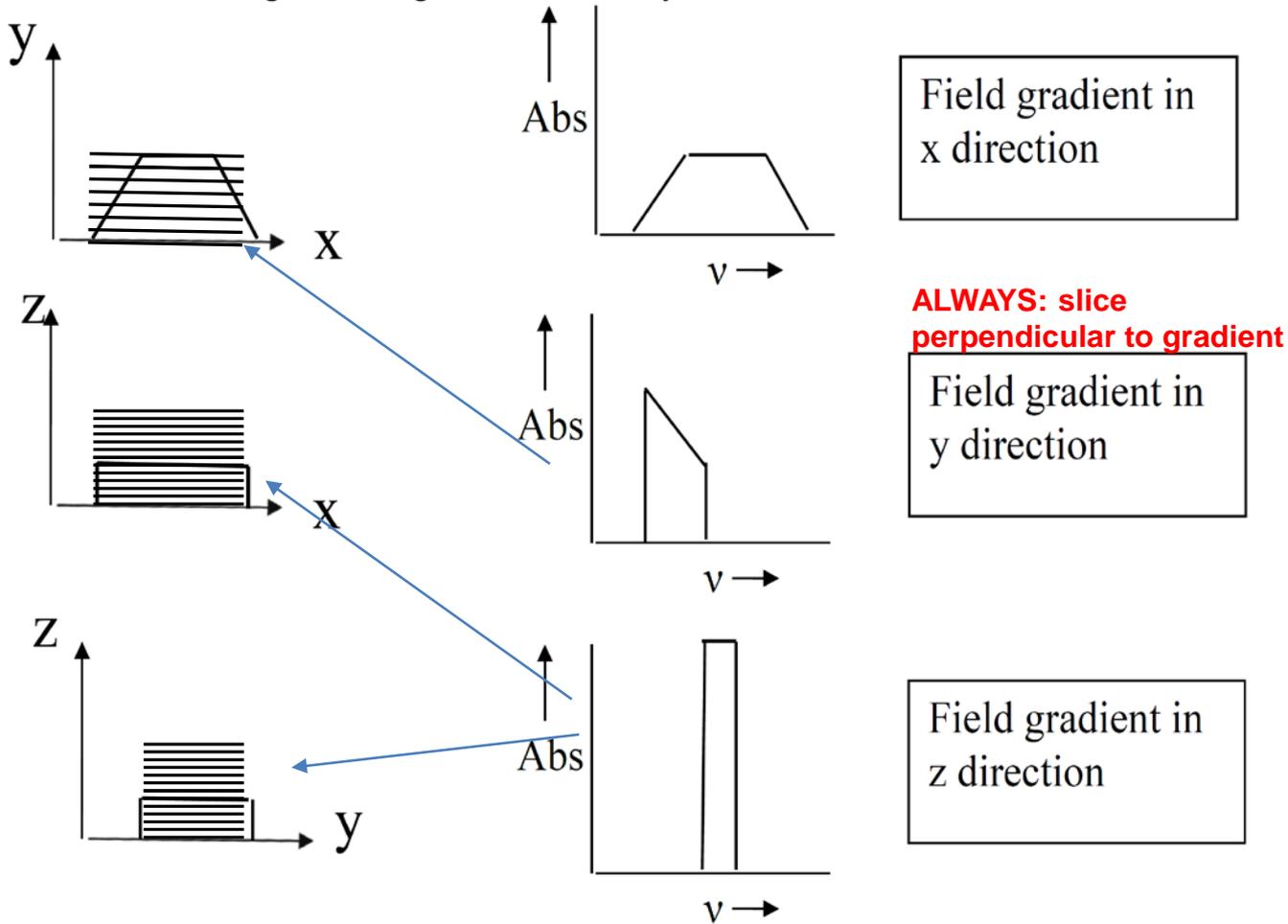


Field gradient in
 y direction



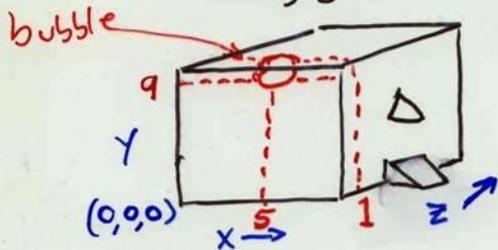
Field gradient in
 z direction

4. Given the shape depicted below shown in 3 views filled with water: draw curves that indicate the intensity of absorbed radio frequency energy during an MRI of this object when the magnetic field gradient is in the x, y, z directions.



MRI - Magnetic Resonance Imaging.

Consider a $10 \times 10 \times 10$ inch "Beaker" of water with "nose", "jaw", and "tumor" ← bubble of air.

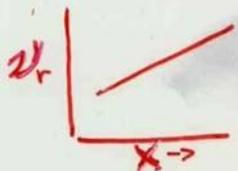
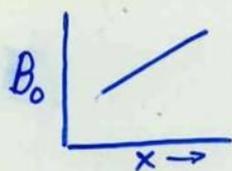


"Tumor" is located at:

$$\begin{aligned} x &= 5 \\ y &= 9 \\ z &= 1 \end{aligned}$$

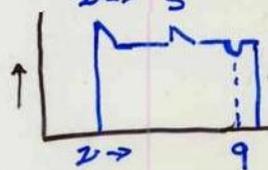
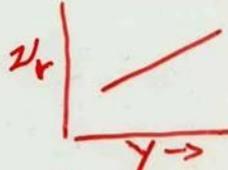
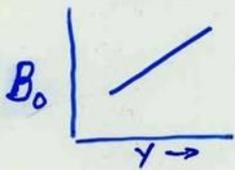
Take 3 spectra with magnetic field gradient $\leftarrow B_0$ along $x, y,$ or z .

Remember, $\nu_r \leftarrow$ resonant frequency for water. proportional to magnetic field strength

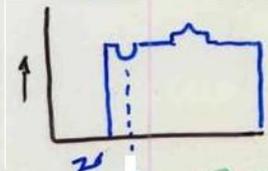
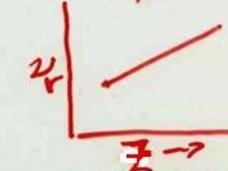
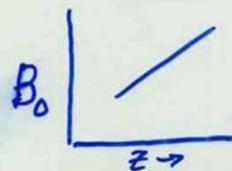


Gradient

X

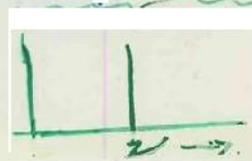
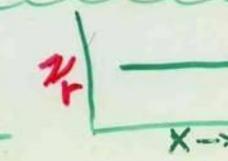
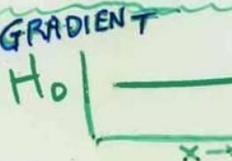


Y



Z

NO GRADIENT



SPIN: internal angular momentum of nuclei and other particles

electron, proton, ^{13}C , ^{15}N , ^{19}F , ^{31}P , ^{17}O : $l = 1/2^+$
 ^2H , ^{14}N : $l = 1$

NOT $\pm 1/2$

* all have magnetic dipoles \neq

The universe is divided into two fundamentally different types of particles:

fermions: have half-integer spin: e.g., electrons, protons, obey **exclusion** principle

bosons: have integer spin e.g., deuterium, photons, obey **inclusion principle** (which means all particles have a tendency to jump into the same state)
 gives phenomena like lasing, superconductivity, etc.

TABLE 14.1 Gyromagnetic Ratios, NMR Frequencies (in an 11.7 T Field), and Natural Abundances of Selected Nuclei

	$\gamma/10^7$ (rad T ⁻¹ s ⁻¹)	ν /MHz	Nat. Abund. /%
^1H	26.75	500.0	99.985
^2H	4.11	76.8	0.015
^{13}C	6.73	125.8	1.108
^{15}N	-2.71	50.6	0.37
^{17}O	-3.63	67.9	0.037
^{19}F	25.18	470.6	100.0
^{29}Si	-5.32	99.5	4.70
^{31}P	10.84	202.6	100.0

$$\text{Boltzmann Constant in } \frac{\text{cm}^{-1}}{\text{K}} = \frac{k_B}{hc} = \frac{1.38 \times 10^{-23} \text{ J/K}}{6.62 \times 10^{-34} \text{ Js } 3 \times 10^{10} \text{ cm}^{-1}} = 0.697 \frac{\text{cm}^{-1}}{\text{K}}$$

Ratio of spin up to spin down:

$$\frac{N_2}{N_1} = e^{-\frac{\Delta E}{k_B T}} = e^{-\frac{\Delta E/hc}{k_B T}} = e^{-\frac{0.02 \text{ cm}^{-1}}{0.697 \text{ cm}^{-1}/\text{K} \cdot 300}} = e^{-\frac{0.02}{207}} = e^{-0.0001} = 0.9999$$

$k_B T$ at 300K = 207 cm^{-1}

This is the same Boltzmann ratio as the change in air pressure from the floor of this room to 1 meter above the floor!!

Ratio of spin up to spin down:

$$\frac{N_2}{N_1} = e^{-\frac{\Delta E}{k_B T}} = e^{-\frac{\Delta E/hc}{\frac{k_B T}{hc}}} = e^{-\frac{0.02 \text{ cm}^{-1}}{0.697 \frac{\text{cm}^{-1}}{\text{K}} \cdot 300}} = e^{-\frac{0.02}{207}} = e^{-0.0001} = 0.9999$$

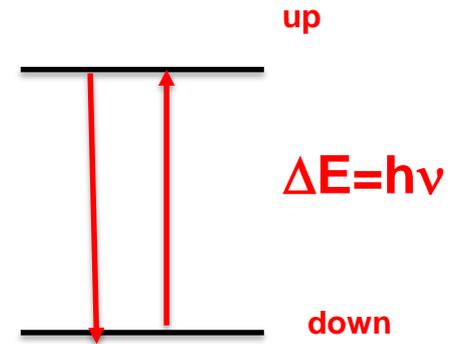
$k_B T$ at 300 K = 207 cm^{-1}

Resonant frequency is RADIO FREQUENCY;

Wavelength is $\sim 1/2$ meter, so

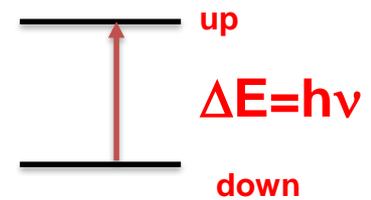
Entire sample literally feels oscillating magnetic field.

Einstein pointed out that excited **spins are stimulated from the high state to low state at same rate as excitation upwards.**



At room temperature only 0.01 % more in lowest state than in highest state

Although there are two quantum “**energy levels**”:
 spin up and spin down, there is a continuous mixture.
 The spins are all in **superposition** states.



$$\Psi_{\text{total}} = C_{\text{up}}\Psi_{\text{up}} + C_{\text{down}}\Psi_{\text{down}}$$

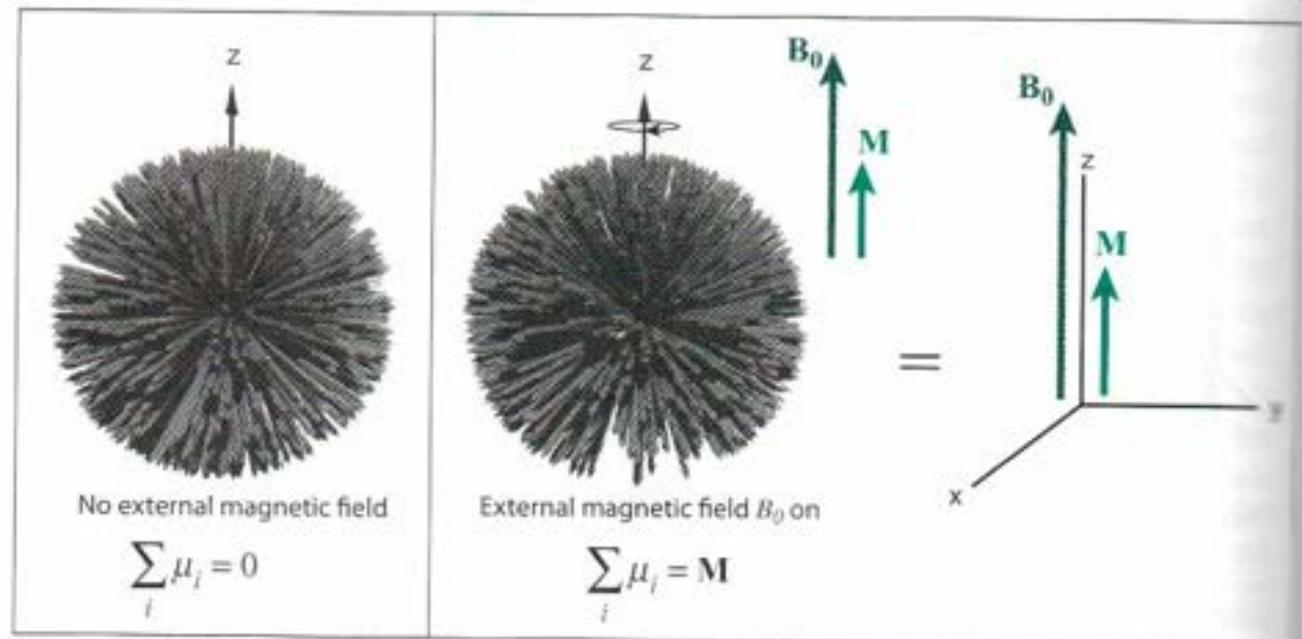
Ψ_{up} and Ψ_{down} are “orbitals” i.e., wavefunctions for the nuclear spin.

The **square** of the coefficients gives probability to observe in the **up** state

$$C_{\text{up}}^2 + C_{\text{down}}^2 = 1$$

The lines pointing in all directions give an idea of the proportion of spin up and spin down for each of the spins.

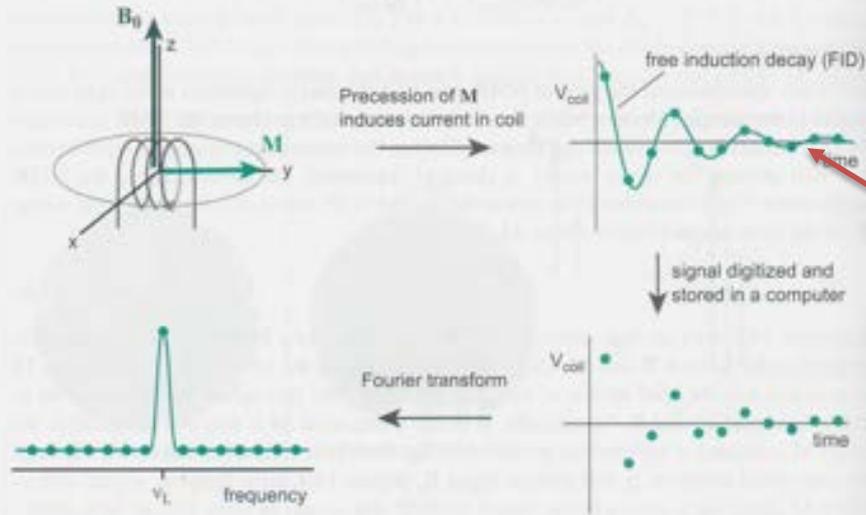
FIGURE 14.3 Visualizing a large number of nuclear magnetic moments of a bulk sample in a bundle. In the absence of any external field (left panel) all nuclear moments μ_i are randomly distributed. In a strong external field along the z-axis (B_0 , right panel), the individual moments are very weakly biased towards the z-axis (the bias is exaggerated 100 fold in the right panel). A vector sum of all of the individual moments reveals the bulk magnetization \mathbf{M} parallel to \mathbf{B}_0 . As we discuss shortly, the individual nuclear moments rotate around B_0 , a motion termed ‘precession.’



$$M_y(t) = M \cos(2\pi\nu t)e^{-t/T_2} \quad (14.7)$$

where T_2 is a time constant governing the loss of the magnetization in the xy -plane. The vector will precess at the Larmor frequency and will return to be aligned along the positive

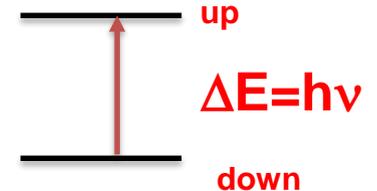
FIGURE 14.5 Signal detection in an NMR experiment. The precessing bulk magnetization \mathbf{M} induces an oscillating current in the coil which may be detected and digitized using electronic test equipment.



intense pulse of radio freq in coil perpendicular changes ratio of up/down from .9999 to 1 Cause coherent motion of the 0.0001 excess spin up. So sample magnet points horizontally. Creates signal in receiver coil (not shown).

Entropy makes sample return to most probable state (Boltzmann distribution). Time to do so is called the **T1 relaxation time**.

We only observe one or the other of the two energy levels when we measure.



Energy is not absolute: Observing the two spin levels in NMR means **only 1 frequency gives resonance**, i.e., will transfer energy to the spins