Homework #1

CHMY 564
11Jan19
Due Wednesday, 16jan19

The statement was made during lecture that the uncertainty principle expression implies quantum mechanical kinetic energy is of the form:

$$E \propto \frac{h^2}{2\mathrm{m}\Delta x^2}$$

While this is obviously true for the particle in an infinite square well (particle in a box) and for the rigid rotor, textbook expressions for the energy of the H atom and one-electron ions are most always expressed in terms a different characteristic length, a , the Bohr radius, and the mass and Planck's constant do not appear:

$$E = -\frac{Z^2}{2n^2}\frac{e^2}{4\pi\varepsilon_0 a}, \text{ in SI units, where } a \text{ stands for the collection of constants } \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

μ is the reduced mass of the nucleus-electron pair, and e is the proton charge.

The harmonic oscillator energy levels are almost always expressed as $E_n=(n+1/2)h\nu$, but can be expressed in terms of a common characteristic length for the harmonic oscillator, the classical turning point in the zero point state, $x_0 = \left(\frac{\hbar}{\mu\omega}\right)^{1/2}$, which is most often seen in the harmonic oscillator wave function as: $\alpha = \left(\frac{\mu\omega}{\hbar}\right) = \frac{1}{x_0^2}$

It is possible to write an exact energy level expression for any system in the form:

$$E \propto \frac{h^2}{2\mathrm{m}\Delta x^2}$$

Write the expression for the zero point energy in this form for the following systems:
(a) particle in a box
(b) rigid rotor
(c) hydrogen atom
(d) harmonic oscillator