The Fock matrix can be seen to be roughly <u>PROPORTIONAL</u> to the Density matrix.

The reason is related to the fact that HF Energy is essentially the "product" of the density and the Fock matrices Notice that element by element, the product $F_{ij} \times P_{ij}$ is negative, with a few exceptions. When the product is positive, this generally indicates antibonding between the two orbitals.

P =	2.108 -0.456 0.000 0.000 -0.104 -0.022 -0.022	2.010 0.000 0.000 0.618 -0.059 -0.059	2.000 0.000 0.000 0.000 0.000	0.737 0.000 0.539 -0.539	1.215 -0.482 -0.482	0.606 -0.183	0.606
F =	-20.236 -5.163 0.000 0.000 0.029 -1.216 -1.216	-2.453 0.000 0.000 0.130 -1.037 -1.037	-0.393 0.000 0.000 0.000 0.000	5) -0.327) 0.000) -0.398) 0.398	-0.353 0.372 0.372	-0.588 -0.403	-0.588

Fock matrix can be seen to roughly PROPORTIONAL to the Density matrix.

Indeed, the entire SCF-HF procedure may said to be equivalent to varying the coefficients that make up the Density Matrix until the density maximally overlaps the Fock matrix.

The reason is related to the fact that HF Energy is essentially the "product" of the density and the Fock matrices.

Notice that element by element, the product $F_{rs} \times P_{rs}$ is negative, with a few exceptions. When the product is positive, this generally indicates antibonding between the two orbitals.

A UNIVERSAL TRUTH:

The Expectation Value of ANY operator A is given by: the <u>trace</u> of the product of the Density Matrix and the operator Matrix

If there is a density matrix, there must be a density operator.

"Density" = Probability Density = $\psi^*\psi$ We can see that in a sense, <A> is the overlap integral of the operator and the density.

$$=\int \Psi^* \hat{A} \Psi d\tau = \int \hat{A} \Psi \(\Psi^*\) d\tau$$

where \hat{A} does not operate on Ψ^*

This is better seen in bra-ket notation:

$$\langle A \rangle = \langle \Psi^* | \hat{A} | \Psi \rangle$$

$$= \langle \Psi^* | \sum_m m \rangle \langle m | \hat{A} | \sum_n n \rangle \langle n | \Psi \rangle$$

$$= \sum_m \sum_n \langle \Psi_i^* | m \rangle \langle m | \hat{A} | n \rangle \langle n | \Psi_i \rangle$$

$$= \sum_m \sum_n c_{\Psi m}^* A_{mn} c_{n\Psi} = \sum_m \sum_n c_{n\Psi} c_{\Psi m}^* A_{mn}$$

$$= \sum_m \sum_n P_{nm} A_{mn} = \sum_n \sum_m P_{nm} A_{mn} = \sum_n (PA)_{nn}$$

= trace PA = sum of diagonal elements of the productof the Density matrix and matrix of operator A

Careful inspection, however, shows that the words used to state this operation *disguise* the underlying simplicity. The operation is indeed *literally* like <u>tracing</u> one matrix on the other.

(Taking the trace of the matrix product is distracting information.) For a symmetric real matrix:

$$\sum_{m} \sum_{n} P_{nm} A_{mn} = \sum_{mn} P_{nm} A_{mn} = \sum_{mn} P_{mn} A_{mn}$$

This is simply the sum of the products of all the corresponding matrix elements, taken in any order. This is the <u>SCALAR PRODUCT</u> of the two matrices, i.e., completely analogous to the overlap integral of the matrices.

a general example

0	1	0	1	0	7	0	1	4	5
1	0	1	0	1	0	6	0	9	0
0	1	0	1	0	1	0	0	0	5
1	0	1	0	1	0	9	0	7	0
0	1	0	1	0	1	8	0	0	0

Once you see the pattern in the left matrix, you can quickly see that the trace of the product of these two matrices is 4 + 8 = 12.

Next we will see how this formalism is hidden in Levine's Eq. 14.45

Physical Interpretation of "MO Energy"

 $\epsilon_{i} = h_{ii} + J_{ii} + sum of 2J_{ij} - K_{ij}$ for all j MOs for j not equal to i

The sum of MO energies counts the electron repulsion TWICE.

Thus, E_{HF} = the sum of MO energies minus the total electron repulsion

$$\sum_{i=1}^{n/2} 2\varepsilon_i = \sum_{i=1}^{n/2} 2h_{ii} + \sum_{i=1}^{n/2} \sum_{j=1}^{n/2} (4J_{ij} - 2K_{ij})$$

So, $E_{HF} = \sum_{i=1}^{n/2} 2\varepsilon_i - \sum_{i=1}^{n/2} \sum_{j=1}^{n/2} (2J_{ij} - K_{ij})$

$$E_{HF} = \frac{1}{2} \sum_{r=1}^{n/2} \sum_{s=1}^{n/2} P_{rs} (F_{rs} + H_{rs}^{core}) + V_{NN}$$
(14.45) Levine

This is the same thing: Instead of subtracting the ee repulsion you ADD the one-electron part with gives twice the E_{HF} then divide all by 2

This shows nicely the trace of the matrix product

Figure 15.2 Shows that increasing the diversity of the basis set lowers The HF-SCF energy considerably, but comes *nowhere near* the exact experimental energy, regardless of the basis set size

	Tartree-Fock Calculatio	ns-			
Reference ^b	Basis Set ^c	Energy/ <i>E</i> h	μ/D	θ	R _{oµ} /Å
CCCBDB	STO-3G, 7	-74.966	1.71	100.0°	0.989
CCCBDB	3-21G, 13	-75.586	2.39	107.7°	0.967
Pitzer, Merrifield	Minimal STO, 7	-75.705	1.92	100.3°	0.990
CCCBDB	6-31G*, 19	-76.011	2.20	105.5°	0.947
CCCBDB	6-31G**, 25	-76.024	2.15	106.0°	0.943
CCCBDB	cc-pVDZ, 24	-76.027	2.04	104.6°	0.946
K. S. Kim et al.	6-311 + +G(2d,2p), 47	-76.057	2.02	106.3°	0.940
CCCBDB	cc-pVTZ, 58	-76.058	1.99	106.0°	0.941
CCCBDB	aug-cc-pVTZ, 92	-76.061	1.93	106.3°	0.941
Dunning et al.	[6s5p2d/3s1p], 43	-76.062	2.08	106.6°	0.941
Rosenberg et al.	(5s4p2d/3s1p) _{STO} , 39	-76.064	2.00	106.1°	0.940
CCCBDB	cc-pVQZ, 115	-76.0655	1.97	106.2°	0.940
CCCBDB	aug-cc-pVQZ, 172	-76.0667	1.94	106.3°	0.940
Amos	[8s6p4d2f/6s3p1d], 112	-76.0675		106.3°	0.940
K. S. Kim et al.	(13s8p4d2f/8s4p2d), 131	-76.0676	1.94	106.3°	0.940
Bakken et al.	cc-pV5Z, 201	-76.0678	1.96	106.33°	0.9396
Bakken et al.	aug-cc-pV5Z, 287	-76.0680	1.94	106.34°	0.9396
Cortez et al.	cc-pV6Z, 322	-76.06810	1.95	106.34°	0.9396
	aug-cc-pV6Z, 443	-76.06815	1.94	106.34°	0.9396
Pahl, Handy	special ^c	-76.06817		106.34°	0.9396
Estimated Hartree-	Fock energy ^d	-76.0682			0.9590
Nonrelativistic fixed	-nuclei energy ^e	-76.438			
Experimental values		-76.480°	1.85	104.5°f	0 9578 ^f

TABLE 15.2	H ₂ O I	Hartree-Fock	Calculations ^a

Estimated Hartree-Fock energyd	-76.068_{3}
Nonrelativistic fixed-nuclei energye	-76.438
Experimental values	-76.480 ^c

This proves that electrons are particles, not clouds!

CCCBDB= Computational Chemistry **Comparison Benchmark Data Base** Other properties get WORSE as basis increased!

Experimental values	-76.480 ^c	1.85	104.5°f	0.9578^{f}
Experimental malana				
Nonrelativistic fixed-nuclei energye	-76.438			
Estimated Hartree-Fock energy ^d	-76.068_{3}			
				0.2020

Energy/ E_h is the total electronic energy including nuclear repulsion in hartrees at the calculated equilibrium geometry; μ , θ , and R_{OH} are the calculated electric dipole moment, equilibrium bond angle, and equilibrium bond length. ⁵CCCBDB is the Computational Chemistry Comparison and Benchmark DataBase (Section 15.1); R. M. Pitzer and D. P. Merrifield, J. Chem. Phys., 52, 4782 (1970); K. S. Kim et al., J. Chem. Phys., 97, 6649 (1992); T. H. Dunning, R. M. Pitzer, and S. Aung, J. Chem. Phys., 57, 5044 (1972); B. J. Rosenberg and I. Shavitt, J. Chem. Phys., 63, 2162 1975) and B. J. Rosenberg, W. C. Ermler, and I. Shavitt, J. Chem. Phys., 65, 4072 (1976); R. D. Amos, J. Chem. Soc. Faraday Trans. 2, 83, 1595 (1987); K. S. Kim et al., op. cit; V. Bakken et al., Mol. Phys., 96, 653 (1999); M. H. Cortez et al., J. Chem. Theory Comput., 3, 1267 (2007); F. A. Pahl and N. C. Handy, Mol. Phys., 100, 3199 (2002). The number of basis functions is given. All the basis sets are GTFs except the Pitzer-Merrifield, Rosenberg et al., and Pahl-Handy sets. Brackets denote CGTF sets and parentheses denote an uncontracted set. A slash separates the oxygen and hydrogen basis functions. The Pahl-Handy set consists of plane waves and radial polynomials.

This estimate is for the calculated Hartree-Fock equilibrium geometry. See D. Feller, C. M. Boyle, and E. R. Davidson, J. Chem. Phys., 86, 3424 (1987).

A. Lüchow, J. B. Anderson, and D. Feller, J. Chem. Phys., 106, 7706 (1997).

P. Jensen et al., J. Mol. Spectrosc., 168, 271 (1994); A. Császár et al., J. Chem. Phys., 122, 214305 (2005).

TABLE 16.1 H₂O Calculations That Include Correlation^a

Reference ^b	Method ^c /Basis Set	Energy/ <u>E_h</u>	$\mu/{\sf D}$	θ	R _{OH} ∔
Harrison, Handy	CISD/DZ	-76.150			
Harrison, Handy	FCI/DZ	-76.158			
CCCBDB	MP2(FC)/6-31G*	-76.197	2.24	104.0°	0.9 69
CCCBDB	CISD(FC)/6-31G*	-76.198	2.24	104.2°	0.966
CCCBDB	MP2(full)/6-31G*	-76.199	2.24	104.0°	0.9 69
CCCBDB	CCSD(FC)/6-31G*	-76.206	2.25	103.9°	0. 969
CCCBDB	MP4(FC)/6-31G*	-76.207	2.25	103.8°	0.970
CCCBDB	CCSD(T)(FC)/6-31G*	-76.208	2.25	103.8°	0. 971
CCCBDB	MP2(FC)/6-31G**	-76.220	2.20	103.8°	0.961
CCCBDB	CISD(FC)/6-31G**	-76.221	2.19	104.2°	0. 958
CCCBDB	MP2(FC)/cc-pVDZ	-76.229	2.10	101. 9°	0.965
CCCBDB	CCSD(T)(FC)/cc-pVDZ	-76.241	2.10	101.9°	0. 966
Scuseria, Schaefer	CISDTQ/DZP	-76.270	2.13	104.5°	0.963
Schaefer et al.	CISD/TZ2P	-76.312	1.94	104.9°	0.952
CCCBDB	CISD(FC)/cc-pVTZ	-76.314	2.03	104.2°	0.953
CCCBDB	MP2(FC)/6-31+G(3df,2p)	-76.318	2.02	104.5°	0. 959
CCCBDB	MP2(FC)/cc-pVTZ	-76.319	2.04	103.5°	0.959
CCCBDB	CCSD(T)(FC)/cc-pVTZ	-76.332	2.04	103.6°	0.9 59
Cortez et al.	CCSD(T)(FC)/cc-pV6Z	-76.372		104.4°	0.958
Kim et al.	$CISD(full)/(13s \dots 2d)^d$	-76.382		104.8°	0.952
Kim et al.	MP2(full)/(13s2d)	-76.391		104.2°	0.959
Kim et al.	CCSD(full)/(13s2d)	-76.396		104.4°	0. 956
Kim et al.	CCSD(T)(full)/(13s2d)	-76.406		104.1°	0. 959
Kim et al.	MP4(full)/(13s2d)	-76.407		104.1°	0. 960
CCCBDB	B3LYP/6-31G*	-76.409	2.10	103.6°	0.969
CCCBDB	B3LYP/6-31G**	-76.420	2.04	103.7°	0.965
CCCBDB	B3LYP/cc-pVDZ	-76.421	1.94	102.7°	0.969
Lüchow et al.	MRCISD/aug-cc-pCV5Z	-76.427 ^e			
Gurtubay, Needs	FN-DQMC	-76.428 ^e			
CCCBDB	B3LYP/cc-pVTZ	-76.460	1.92	104.5	0. 961
Nonrelativistic fixed-	nuclei energy	-76.438			
Experimental values		-76.480	1.85	104.5°	0.958

^aSee footnote a to Table 15.2 in Section 15.6,

^bR. J. Harrison and N. C. Handy, *Chem. Phys. Lett.*, **95**, 386 (1983); cccbdb.nist.gov; G. E. Scuseria and H. F. Schaefer, III, *Chem. Phys. Lett.*, **146**, 23 (1988); H. F. Schaefer, III et al., in *Yarkow*, Chapter 1; M. H. Cortez et al., *J. Chem. Theor. Comput.*, **3**, 1267 (2007); J. Kim et al., *J. Chem. Phys.*, **102**, 310 (1995); A. Lüchow, J. B. Anderson, and D. Feller, *J. Chem. Phys.*, **106**, 7706 (1997); I. G. Gurtubay and R. J. Needs, *J. Chem. Phys.*, **127**, 124306 (2007).

°FC denotes frozen-core calculations and full indicates that the frozen-core approximation is not used.

 $d(13s \dots 2d)$ is an uncontracted (13s8p4d2f/8s4p2d) set with 131 basis functions.

eCalculated at a geometry close to the experimental geometry.

If electron-correlation is added to the wavefunction, <u>all</u> properties converge to experimentally observed values.

CCCBDB	B3LYP/6-31G*	-76.409	2.10	103.6°	0.9 69
CCCBDB	B3LYP/6-31G**	-76.420	2.04	103.7°	0. 965
CCCBDB	B3LYP/cc-pVDZ	-76.421	1.94	102.7°	0.969
Lüchow et al.	MRCISD/aug-cc-pCV5Z	-76.427 ^e			
Gurtubay, Needs	FN-DQMC	-76.428 ^e			
CCCBDB	B3LYP/cc-pVTZ	-76.460	1.92	104.5	0. 961
Nonrelativistic fixed-nuclei energy		-76.438			
Experimental values		-76.480	1.85	104.5°	0. 958

^aSee footnote a to Table 15.2 in Section 15.6.

^bR. J. Harrison and N. C. Handy, *Chem. Phys. Lett.*, **95**, 386 (1983); cccbdb.nist.gov; G. E. Scuseria and H. F. Schaefer, III, *Chem. Phys. Lett.*, **146**, 23 (1988); H. F. Schaefer, III et al., in *Yarkony*, Chapter 1; M. H. Cortez et al., *J. Chem. Theor. Comput.*, **3**, 1267 (2007); J. Kim et al., *J. Chem. Phys.*, **102**, 310 (1995); A. Lüchow, J. B. Anderson, and D. Feller, *J. Chem. Phys.*, **106**, 7706 (1997); I. G. Gurtubay and R. J. Needs, *J. Chem. Phys.*, **127**, 124306 (2007). ^CFC denotes frozen-core calculations and full indicates that the frozen-core approximation is not used. ^d(13s...2d) is an uncontracted (13s8p4d2f/8s4p2d) set with 131 basis functions.

eCalculated at a geometry close to the experimental geometry.

B3LYP is the most popular DFT method; B3LYP/cc-pVTZ 99.87% of experiment; Takes 5 sec on a PC

5.2 A BIT OF THEORY: SLATER VS GAUSSIAN

Both Slater Type Orbitals (STOs) and Gaussian Type Orbitals (GTOs) are used to describe AOs. STOs describe the shape of AOs more closely than GTOs, but GTOs have an unbeatable advantage: they are much easier to compute. In fact, it is faster to compute several GTOs and combine them to describe an orbital than to compute one STO! This is why combinations of GTOs are commonly used to describe STOs, which in turn, describe AOs. Yes, a bit complicated, but the computers don't mind. There are other differences between STOs and GTOs, but they will not be covered here.



Figure 6: Slater VS Gaussian Type functions [4]

The usual Spherical Harmonics

Slater Type Orbital (STO) = $N r^{n-1} e^{-\zeta r/a_0} Y_l^m(\theta, \phi)$

Note: No spherical nodes in Slater Type functions

Alpha	Αα	Iota	Iι	Rho	Ρρ
Beta	Bβ	Карра	Kκ	Sigma	Σσ
Gamma	Γγ	Lambda	$\Lambda \lambda$	Tau	Ττ
Delta	$\Delta \delta$	Mu	Μμ	Upsilon	Yυ
Epsilon	Εε	Nu	Nν	Phi	Φφ
Zeta	Zζ	Xi	E) الح	Chi	Хχ
Eta	Ηη	Omicron	O o	Psi	$\Psi\psi$
Theta	Θθ	Рi	Ππ	Omega	$\Omega \omega$

www.1728.com

Basis Sets

http://gaussian.com/glossary/g09/ http://gaussian.com/basissets/

For all methods, a choice of basis set must be done: $MO_i = \phi_i =$ linear combination of STOs or Gaussian type functions (GTFs) (LC-STO, LC-GTF)

This includes Density Functional Theory (DFT) in which one does not have a wave function, yet works with Kohn-Sham orbitals, which are a fictitious reference system of non-interacting electrons <u>of no physical significance</u> that allow one to construct the exact density-- (*if and when someone learns how*). There is no wavefunction in DFT.

Diatomic basis functions are STOs (not so many e-e repulsion integrals)

Typically use **G**aussian **T**ype **F**unctions for non-linear larger triatomic and larger linear combinations of LC-GTF, because of the large number of e-e repulsion integrals that are very time-consuming for STOs.

a "primitive" **GTF** on atom **b**:

$$g_{ijk} = N x_b^i y_b^j z_b^k e^{-\alpha r_b^2}$$

If i + j + k = 0, it is an s GTF
If i + j + k = 1, it is a p GTF only 3
If i + j + k = 2, it is a d GTF typically 6 (6 ways i+j+k = 2)
If i + j + k = 3, it is an f GTF typically 10 (10 ways i+j+k = 3)
etc.

e.g., the 6 d functions are : x^2 , y^2 , z^2 , xy, xz, yzused to seeing 5 = xy, xz, yz, x^2-y^2 , $r^2-3z^2 = x^2 + y^2 - 2z^2$ i.e., "d $z^{2"}$ which are all linear combinations of the 6 GTFs

The 6th wheel is $x^2 + y^2 + z^2 = r^2$, i.e., another s type (not d)

Requires more GTFs than STOs, but no 4-center e-e repulsion (rs|tu) to calculate because product of 2 Gaussians is another Gaussian centered in at different place.

In practice contracted GTFs (CGTFs) are used as the basis functions.

A CGTF =
$$\chi_{\mu} = \sum_{p} d_{\mu p} g_{p}$$
 MOi = $\varphi_{i} = \sum_{\mu} c_{\mu i} \chi_{\mu}$

and the $d_{\mu\rho}$ are constants previously optimized for the given basis set and never change.

Some TERMS

minimal basis e.g., STO-3g

double zeta DZ (DZ stands for double zeta; what's zeta? ζ Often used as the orbital exponent instead of α); **DZ means two different sizes of everything; TZ =triple zeta= 3 different sizes, etc.**

quadruple QZ 5Z 6Z

split valence VDZ, VTZ, etc. inner shells are typically a single function but valence shell gets multiple zeta

Examples: 3-21g : inner shells 1 contracted GTF (CGTF) made of 3 primitive GTFs Valence shell is double zeta: 1 CGTF made of 2 primitive GTFs and one made of 1 primitive GTF

6-31g : inner shells 1 contracted GTF (CGTF) made of 6 primitive GTFs Valence shell is double zeta: 1 CGTF made of 3 primitive GTFs and one made of 1 primitive GTF

More TERMS

Polarization Basis sets

The previous page was about changing the size of orbitals without change in shape. But when in a molecule, the electron density on an atom is not the same as an isolated atom. This is accomplished by adding orbitals that have d character in the first row, and p character for H.

Examples: 6-31g* = 6-31g(d) adds 6 d-type GTFs for the atoms Li-Ca, and 10 f-type GTFs for Sc-Zn.

6-31g^{**} = 6-31g(d,p) adds p character to H, in addition.

Diffuse Basis Sets

Anions particularly benefit from include large basis functions. e.g, 6-31+g includes extra large s and p functions on heavy atoms, and 6-31++g in addition at a diffuse s orbital to H

Even Larger Basis sets

For example 6-311++G(3df,3pd) is triple zeta set defined for H-Ar, that provides diffuse functions on all atoms, 3 sets of d and 1 set of f functions to non-H atoms, and 3 sets of p functions, 1 set of d functions for H. This also called a VTZ3PD, meaning valence triple zeta with triple polarization and diffuse on all atoms.

The above were developed by Pople.

Dunning and co-workers developed as series of basis functions denoted cc-pVnZ: i.e., cc-pVDZ, cc-pVTZ...,cc-pV6Z, which stand for *correlation-consistent*, *polarized valence double-zeta*, etc.

These basis sets were specifically designed to be used with methods that introduce electron correlation. Each set increases the orbital types as follows:

cc-pVDZ,	cc-pVTZ	cc-pVQZ ,	cc-pV5Z	cc-pV6Z
[3s2p1d]	[4s3p2d1f]]	[5s4p3d2f1g]	[6s5p4d3f2g	1h] [7s6p5d4f3g2h1i]

The number and degree of polarization (angular momentum) function increases rapidly from left to right, meaning <u>more nodes</u>, which is what is needed to introduce fine-grained correlation (better dodging). Doing a series of calculations with these basis sets on a molecule allow smooth extrapolation to what may be considered a near exact value.

If diffuse functions are added to the cc-pVnZ sets, they are called **aug- cc-pVnZ**.

Some General Terms

vibrational frequency (must be opt first)

<u>Names</u>

method/basis

geometry optimization

transition state search

reaction path following

potential energy surface

single point

solvation

for example:

http://gaussian.com/keywords/

key words	
hf/6-31g*	
ont	
opt freq	

Open Shell Calculations (some MOs have only 1 electron)

ROHF (restricted open shell) means electrons with opposite spin in same MO have the same spatial orbital.

UHF (unrestricted HF) means alpha and beta spins may have different spatial functions, thereby giving a slightly lower energy. (but, will not usually be eigenfunctions of the S² operator (i.e., neither singlet or triplet, for example)