The statement was made during the previous lecture that all energy level expressions from quantum mechanics can be expressed in the form:

$$E \propto \frac{\hbar^2}{2m\Delta x^2}$$

While this is obviously true for the particle in an infinite square well (particle in a box) and for the rigid rotor, textbook expressions for the energy of the H atom and one-electron ions are most always expressed as

$$E = -\frac{Z^2}{2n^2} \frac{e^2}{4\pi \epsilon_0 a}$$

in SI units, where \(a\) stands for the collection of constants \(\frac{4\pi \epsilon_0 \hbar^2}{\mu e^2}\)

\(\mu\) is the reduced mass of the nucleus-electron pair, and \(e\) is the proton charge.

In class we found a characteristic length for the harmonic oscillator to be the classical turning point in the zero point state, \(x_0 = \left(\frac{\hbar}{\mu \omega}\right)^{1/2}\), which is most often seen in the harmonic oscillator wave function as: \(\alpha = \left(\frac{\mu \omega}{\hbar}\right) = \frac{1}{x_0^2}\)

Write the expression for the zero point energy of the following systems in the form:

$$E \propto \frac{\hbar^2}{2m\Delta x^2}$$

(a) particle in a box
(b) rigid rotor
(c) hydrogen atom
(d) harmonic oscillator